

RZWQM2 Phosphorus Model

Technical Report

Documentation Developed by

Debasis Sadhukhan

Zhiming Qi

Department of Bioresource Engineering

McGill University

Sainte-Anne-de-Bellevue

Quebec, H9X 3V9, Canada

May 1, 2018



McGill

Table of Content

PARTICULARS	Page No
1. Initialization of P Pools	1
2. Fertilizer P Dynamics	1
3. Manure P Dynamics	4
4. Soil P Dynamics	10
5. Plant P Uptake	13
6. P flow from Residue and Soil Organic matter	15
7. Tillage	16
8. DRP loss in Runoff	17
9. PP loss in Runoff	18
10. DRP loss in tile drainage	18
11. PP loss in tile drainage	20
12. References	24

1. Initialization of P Pools

Before the start of model simulations, the soil P pools need to be initialized. Initial amount of labile P pool and stable organic need to be specified by the model user. Other P pools are initialized as follows.

Active P Pool is initialized as:

$$ActP = Labp * \frac{1-PSP}{PSP} \quad (1)$$

Where, ActP = Active P amount in a soil layer (Kg/ha)

Labp = Labile P amount in a soil layer (Kg/ha)

PSP = Phosphorus sorption coefficient (or P availability index) (Williams et al., 2008)

PSP is calculated as follows

$$PSP = -0.045 * \text{Log}(\text{Clay}) + 0.001 * \text{LabP} - 0.035 * \text{SoilOC} + 0.43 \quad (2)$$

Where, Clay = Clay % of soil

Labp = Labile P amount in a soil layer (mg/Kg)

SoilOC = Soil Organic Carbon (%)

$$\text{SoilOC} = \text{SoilOM} * 0.58 \quad (3)$$

Where, SoilOM = Soil organic matter in a layer (%)

Inorganic Stable P pool is initialized as:

$$\text{StabiP} = 4 * \text{ActP} \quad (4)$$

Where, StabiP = Stable Inorganic P pool in a soil layer. (Kg/ha)

ActP = Active P amount in a soil layer (Kg/ha)

Fresh organic P pool in a soil layer is set to 0.03% of initial amount of crop residue present in that a layer respectively.

$$\text{Frsop} = 0.0003 * \text{Crpres} \quad (5)$$

Where, Frsop = Fresh organic P pool in a soil layer (Kg/ha).

Crpres = Crop residue amount in a soil layer (Kg/ha)

All the surface manure and fertilizer P pool are initialized as zero.

2. Fertilizer P Dynamics

Model assumes that when a fertilizer is applied the fertilizer P is instantaneously is divided between two surface fertilizer pools based on depth of application namely available fertilizer P pool and residual fertilizer P pool. 75% of fertilizer P is added to available fertilizer pool and 25% is added to residual fertilizer pool (Vadas, 2014; Williams, 1969). The P in the available

fertilizer pool is readily available to be lost by runoff and to be adsorbed by soil. The adsorbed fertilizer P is added to the soil labile P pool of the first soil layer.

So, in case of surface applied fertilizer i.e. depth of application is zero

$$AvfertP = 0.75 * FertP \quad (6)$$

$$ResfertP = 0.25 * FertP \quad (7)$$

And in case of subsurface application

$$AvfertP = 0.75 * Fertp * Fsurf \quad (8)$$

$$ResfertP = 0.25 * Fertp * Fsurf \quad (9)$$

Where, Avfertp = Available fertilizer P pool (Kg)

Resfertp = Residual fertilizer P Pool (Kg)

FertP = Fertilizer P applied in the field (Kg)

Fsurf = Fraction of fertilizer left on surface during application (-)

The fraction of FertP which is applied below soil surface is directly added to the soil labile P pool depending upon a factor based on the ratio of soil layer thickness to fertilizer application depth.

For a layer having depth less than fertilizer application depth

$$Fact = \frac{Tsoil}{FertD} \quad (10)$$

For the soil layer where the fertilizer is applied

$$Fact = 1 - \sum_1^k Fact_k \quad (11)$$

$$LabP_a = LabP_b + FertP * Fact * (1 - Fsurf) \quad (12)$$

Where, Fact = A factor (-)

Tsoil = Thickness of a soil layer (m)

FertD = Fertilizer application depth (m)

LabP_b = Labile P pool of a soil layer before subsurface fertilizer application. (Kg)

LabP_a = Labile P pool of a soil layer after subsurface fertilizer application. (Kg)

k = Number of soil layers having depth less than fertilizer application depth. (-)

After the fertilizer application, once the rainfall happens all the P in Avfertp pool is released. For the second rainfall, 40% of the P in Resfertp is released and from the third rainfall onwards, consistently about 7.5% of the remaining P in Resfertp was released until all the P in Resfertp pool is exhausted (Vadas, et al., 2008).

So, for the case of first rainfall event

$$Fertprelease = AvfertP \quad (13)$$

For the case of 2nd rainfall event

$$Fertprelease = 0.40 * ResfertP \quad (14)$$

For the case of 3rd rainfall onwards

$$Fertprelease = 0.075 * ResfertP \quad (15)$$

Where, Fertprelease = Amount of P is released due to rainfall from the fertilizer P pools. (Kg)

This released P is either lost through runoff or absorbed in soil labile P pool with infiltration or both depending upon a factor based on rainfall and runoff (Vadas, 2014; Vadas et al., 2008). Factor, that represents the distribution of released fertilizer P between runoff and infiltration and is calculated as

$$PDFACTOR = 0.034 * e^{(3.4 * \frac{Runoff}{Rainfall})} \quad (16)$$

Where, PDFACTOR = P distribution factor

Runoff = Runoff amount (cm)

Rainfall = Rainfall amount (cm)

So, P concentration in runoff water due to loss of P from fertilizer P pool

$$FertPcrunoff = \frac{Fertprelease * PDFACTOR}{Rainfall * Area * 100} \quad (17)$$

Where, FertPcrunoff = P concentration in runoff water due to loss of P from fertilizer P pool (Kg/m³)

Area = Area of the field (ha)

P mass in in runoff water due to loss of P from fertilizer P pool is calculated as

$$FertPmrutoff = FertPcrunoff * Runoff * Area * 100 \quad (18)$$

Where, FertPmrutoff = P mass in in runoff water due to loss of P from fertilizer P pool (Kg)

The amount of P mass that is released but not carried away by runoff is adsorbed to the soil labile pool as follows

$$LabP_a(1) = LabP_b(1) + (Fertprelease - FertPmrutoff) * 0.8 \quad (19)$$

$$LabP_a(2) = LabP_b(2) + (Fertprelease - FertPmrutoff) * 0.2 \quad (20)$$

Where, LabP_a (1) = Labile P pool of first soil layer after adsorption. (Kg)

LabP_b (1) = Labile P pool of first soil layer before adsorption. (Kg)

LabP_a (2) = Labile P pool of second soil layer after adsorption. (Kg)

LabP_b (2) = Labile P pool of second soil layer before adsorption. (Kg)

In between, the first rainfall event and fertilizer application, the P in the Avfertp is being absorbed in the soil and added to soil labile P pool of the first soil layer. But this absorption rate

varies according to the land cover type. It does at a slower rate for grassed or residue-covered soils than for bare soils (Vadas et al., 2008; Williams, 1969). The equations used to calculate the fraction of applied fertilizer P that remains available on the soil surface over time after application are

For bare soil:

$$Fertpfr = -0.16 * \ln(Days) + 0.65 \quad (21)$$

For residue covered soil:

$$Fertpfr = -0.16 * \ln(Days) + 0.75 \quad (22)$$

From crop covered soil:

$$Fertpfr = -0.16 * \ln(Days) + 0.85 \quad (23)$$

Where, Fertpfr = Fraction of P in AvFert pool remaining after absorption. (-)

Days = Number of days since application. (Days)

3. Manure P Dynamics

To simulate manure P dynamics user need to specify the day of manure application, the percentage of field covered, the % percentage of manure left on surface during application (100% for surface application, 0% for total sub-surface application), mass of manure applied, manure dry matter content (%), total P content (TP) (%), water extractable inorganic P content (%), water extractable organic P content (%). At the day of manure application, the applied manure P is divided into four surface manure P pools based on total P content, water extractable inorganic P content, water extractable organic P content, type of application i.e. weather surface or subsurface, and type of manure i.e. weather liquid or solid. In case of liquid manure i.e. the manure with dry matter content less than 15%, model assumes that 60% of manure P immediately infiltrates into soil and added the respective soil active and labile P pools (Vadas et al., 2004; P. Vadas et al., 2006). At the time of manure application, the manure P is distributed in four surface manure P pools namely manure water extractable inorganic P pool, manure water extractable organic P pool, manure stable inorganic P pool, manure stable organic P pool. Water extractable P pools represent P that can be released from manure by rain and stable P pools represents P that can be released by rain but can be transformed to water extractable pools as manure decomposes and mineralizes (Vadas et al., 2007). The size of the water extractable inorganic and organic P pool are determined based on the percentage of water extractable inorganic P and percentage of water extractable organic P present in the manure. The difference between manure total P and sum of water extractable inorganic P and water extractable organic P is the stable P. The model divides this stable P into inorganic and organic P pools according to 25/75 ratio (Ajiboye et al., 2004; Dou, et al., 2000; He, et al., 2003; He and Honeycutt., 2001; He et al., 2006; McDowell et al., 2005; McGrath et al., 2005; Turner et al., 2004; Vadas, 2014).

$$Manwip = (Manpmass * S * L) * \frac{Manweipper}{100} \quad (24)$$

$$Manwop = (Manpmass * S * L) * \frac{Manweopper}{100} \quad (25)$$

$$Mansop = (Manpmass * S * L) * \left(1 - \frac{Manweipper}{100} - \frac{Manweipper}{100}\right) * 0.75 \quad (26)$$

$$Mansip = (Manpmass * S * L) * \left(1 - \frac{Manweipper}{100} - \frac{Manweipper}{100}\right) * 0.25 \quad (27)$$

Where, Manwip = Manure water extractable inorganic P Pool. (Kg)

Manwop = Manure water extractable organic P Pool. (Kg)

Mansip = Manure stable inorganic P pool. (Kg)

Mansop = Manure stable organic P pool. (Kg)

Manpmass = Manure P mass applied. (Kg)

Manweipper = Percentage of water extractable inorganic P (%)

Manweopper = Percentage of water extractable organic P (%)

S = fraction of manure P mass left on surface during application (1 for surface application, 0-1 for subsurface application).

L = Fraction of manure P mass stay on surface after infiltration of manure P into the soil during application (0.4 for liquid manure, 1 for solid manure)

In case of liquid manure, the 60% manure P is absorbed to the soil active and labile P pool as follows

$$ActP_a(1) = ActP_b(1) + (Manpmass * S * (1 - L)) * \left(1 - \frac{Manweipper}{100} - \frac{Manweipper}{100}\right) * 0.25 \quad (28)$$

$$\begin{aligned} LabP_a(1) = & LabP_b(1) + (Manpmass * S * (1 - L)) * \frac{Manweipper}{100} \\ & + (Manpmass * S * (1 - L)) * \frac{Manweopper}{100} * 0.95 \\ & + (Manpmass * S * (1 - L)) * \left(1 - \frac{Manweipper}{100} - \frac{Manweipper}{100}\right) * 0.75 * 0.95 \end{aligned} \quad (29)$$

$$\begin{aligned} LabP_a(2) = & LabP_b(2) + (Manpmass * S * (1 - L)) * \frac{Manweopper}{100} * 0.05 \\ & + (Manpmass * S * (1 - L)) * \left(1 - \frac{Manweipper}{100} - \frac{Manweipper}{100}\right) * 0.75 * 0.05 \end{aligned} \quad (30)$$

Where, LabP_a (1) = Labile P pool of first soil layer after absorption. (Kg)

LabP_b (1) = Labile P pool of first soil layer before absorption. (Kg)

LabP_a (2) = Labile P pool of second soil layer after absorption. (Kg)

LabP_b (2) = Labile P pool of second soil layer before absorption. (Kg)

ActP_a (1) = Active P pool of first soil layer after absorption. (Kg)

ActP_b (1) = Active P pool of first soil layer before absorption. (Kg)

In case of sub-surface application of manure, the manure P which is applied below ground surface is directly added to soil labile P and active P pool depending upon a factor based on the ratio of soil layer thickness to fertilizer application depth as follows.

For a layer having depth less than manure application depth

$$Fact = \frac{T_{soil}}{ManD} \quad (31)$$

For the soil layer where the fertilizer is applied

$$\begin{aligned} Fact &= 1 - \sum_1^k Fact_k \quad (32) \\ LabP_a &= LabP_b + (Manp_{mass} * (1 - S)) * \frac{Man_{weipper}}{100} * Fact \\ &+ (Manp_{mass} * (1 - S)) * \frac{Man_{weipper}}{100} * Fact \\ &+ (Manp_{mass} * (1 - S)) * \left(1 - \frac{Man_{weipper}}{100} - \frac{Man_{weipper}}{100}\right) * 0.75 * Fact \quad (33) \end{aligned}$$

$$ActP_a = ActP_b + (Manp_{mass} * (1 - S)) * \left(1 - \frac{Man_{weipper}}{100} - \frac{Man_{weipper}}{100}\right) * 0.25 * Fact \quad (34)$$

Where, Fact = A factor (-)

T_{soil} = Thickness of a soil layer (m)

ManD = Manure application depth (m)

LabP_b = Labile P pool of a soil layer before subsurface manure application. (Kg)

LabP_a = Labile P pool of a soil layer after subsurface manure application. (Kg)

ActP_a = Active P pool of a soil layer after subsurface manure application. (Kg)

ActP_b = Active P pool of a soil layer before subsurface manure application. (Kg)

k = Number of soil layers having depth less than manure application depth. (-)

After manure application, as the manure ages, manure and P in the Mansip, Mansop, Manwop Pool decomposes and assimilates based on ambient temperature and manure moisture content (Vadas, 2014).

Daily manure decomposition rate is calculated as

$$Mandcomr = 0.003 * TFA^{0.5} \quad (35)$$

Where, Mandcomr = Manure decomposition rate (per day)

TFA = Unit less temperature factor (-). Varies between 0-1.

TFA depends on daily atmospheric temperature and it is calculated as

$$TFA = \frac{2*32^2*T^2 - T^4}{32^4} \quad (36)$$

Where, T = Average daily atmospheric temperature ($^{\circ}\text{C}$)

Daily manure assimilation rate is calculated as

$$Manasimr = 30.0 * e^{(2.5*Moist)} \quad (37)$$

Where, Manasimr = Manure assimilation rate (per day)

Moist = Unit less Manure moisture content factor (-). Varies between 0-0.9

Moist depends on amount of rainfall and it is calculated as

If no rainfall i.e. rainfall amount = 0

$$Moist = Moist_0 - \left(0.075 - 0.05 * \frac{ManMass}{Applied\ ManMass}\right) * TFA \quad (38)$$

If rainfall is less than 4 mm

$$Moist = Moist_0 \quad (39)$$

If rainfall is more than 4 mm

$$Moist = Moist_0 + (0.27 - 0.3 * Moist_0) \quad (40)$$

Where, Moist₀ = Manure moisture factor of the previous day. (-)

TFA = Temperature factor. (-)

ManMass = Current manure mass present in the field. (Kg)

Applied ManMass = Initial amount of manure applied in the field. (Kg)

Manure decomposition is calculated as

$$Mandcom = ManMass * Mandcomr \quad (41)$$

Where, Mandcom = Manure decomposition. (Kg/day)

Manure coverage area also decomposes at a same ratio as manure decomposes

$$Mancovadcom = \frac{Mandcom}{Manmass} * Mancov \quad (42)$$

Where, Mancovadcom = Manure cover area decomposition (ha/day)

Mancov = Manure Cover area (ha)

Decompositions of P from Mansop, Mansip, Manwop Pool is calculated as

$$Mansopdcom = 0.01 * Mansop * MIN(TFA, Moist) \quad (43)$$

$$Mansipdcom = 0.0025 * Mansip * MIN(TFA, Moist) \quad (44)$$

$$Manwopdcom = 0.1 * Manwop * MIN(TFA, Moist) \quad (45)$$

Where, Mansopdcom = Manure stable organic P decomposition. (Kg/day).

Mansipdcom = Manure stable inorganic P decomposition. (Kg/day).

Manwopdcom = Manure water extractable organic P decomposition. (Kg/day).

Mansop = Manure stable organic P pool. (Kg)

Mansip = Manure stable inorganic P pool. (Kg)

Manwop = Manure water extractable P pool. (Kg)

75% of the decomposed P from Mansop is added to Manwip pool and remaining 25% is added to Manwop pool. All the decomposed Mansip and Manwop pool is added to Manwip pool (McGrath et al., 2005).

Manure assimilation is calculated as

$$Manasim = Manasimr * TFA * Mancov \quad (46)$$

Manure cover area and Manure P pools are assimilated at the same ratio as the manure mass assimilate as follows

$$Mancovasim = \frac{Manasim}{Manmass} * Mancov \quad (47)$$

$$Manwipasim = \frac{Manasim}{Manmass} * Manwip \quad (48)$$

$$Manwopasim = \frac{Manasim}{Manmass} * Manwop \quad (49)$$

$$Mansipasim = \frac{Manasim}{Manmass} * Mansip \quad (50)$$

$$Mansopasim = \frac{Manasim}{Manmass} * Mansop \quad (51)$$

Where, Mancovasim = Manure cover area assimilation. (ha/day)

Manwipasim = Manwip pool assimilation (Kg/day)

Manwopasim = Manwop pool assimilation (Kg/day)

Mansipasim = Mansop pool assimilation (Kg/day)

Mansopasim = Mansip pool assimilation (Kg/day)

Assimilated P is added to the soils labile and active P pools. 60% of assimilated P is added to the respective P pool of the first soil layer. If the depth of the 2nd layer is less than 15 cm then 30% of it is added to the respective P Pools 2nd soil layer and rest 10% is added the respective P pools of to the 3rd soil layer. If the depth of the 2nd layer is more than 15 cm then 40% of assimilated P is added to the respective P pool of the 2nd soil layer (Vadas, 2014).

$$ActP_a(1) = ActP_b(1) + Mansipasim * 0.6 \quad (52)$$

$$LabP_a(1) = LabP_b(1) + (Manwipasim + Manwopasim + Msopasim) * 0.6 \quad (53)$$

If the depth of the second layer is more than 15 cm then

$$ActP_a(2) = ActP_b(2) + Mansipasim * 0.4 \quad (54)$$

$$LabP_a(2) = LabP_b(2) + (Manwipasim + Manwopasim + Msopasim) * 0.3 \quad (55)$$

If the depth of the second layer is less than 15 cm then

$$ActP_a(2) = ActP_b(2) + Mansipasim * 0.3 \quad (56)$$

$$ActP_a(3) = ActP_b(3) + Mansipasim * 0.1 \quad (57)$$

$$LabP_a(2) = LabP_b(2) + (Manwipasim + Manwopasim + Msopasim) * 0.3 \quad (58)$$

$$LabP_a(3) = LabP_b(3) + (Manwipasim + Manwopasim + Msopasim) * 0.1 \quad (59)$$

After daily manure assimilation and decompositions the Manure mass, Manure coverage area and manure P pools are updated as follows

$$Manmass_a = Manmass_b - Mandcom - Manasim \quad (60)$$

$$Mancov_a = Mancov_b - Mancovdcom - Mancovasim \quad (61)$$

$$Mansip_a = Mansip_b - Mansipdcom - Mansipasim \quad (62)$$

$$Mansop_a = Mansop_b - Mansopdcom - Mansopasim \quad (63)$$

$$Manwop_a = Manwop_b - Manwopdcom - Manwopasim + Mansopdcom * 0.25 \quad (64)$$

$$Manwip_a = Manwip_b - Manwipasim + Manwopdcom + Mansopdcom * 0.75 + Mansipdcom \quad (65)$$

a,b stand for manure mass , coverage are and pool sizes after and before a particular day respectively.

When rainfall occurs, P from manure water extractable pools is released. This released P is either carried away by runoff or absorbed in soil labile P pool of the first soil layer. The amount of P release depends on rainfall amount and rain to manure mass ratio (Vadas et al., 2005; Vadas et al., 2004). Amount of P release from manure water extractable pools is released is calculated as

$$Manprelease = Manextrc * (Manwip + Manwop) \quad (66)$$

Where, Manprelease = Manure P release due to rainfall (Kg /day)

Manextrc = Manure extraction coefficient (Per day). It value varies between 0-1

If no rainfall then Manextrc = 0.

Manextrc is calculated as

For dairy and beef manure

$$Manextrc = \frac{1.2*W}{W+73.1} \quad (67)$$

For Poultry and swine manure

$$Manextrc = \frac{2.2*W}{W+300.1} \quad (68)$$

Where, W is rain to manure mass ratio (cm^3/gm) and is calculated as

$$W = \frac{\text{Rain}}{\text{Manmass}} * \text{Mancov} * 10^5 \quad (69)$$

Where, Rain = Amount of rainfall. (cm)

Manmass = Manure mass. (Kg)

Mancov = Manure coverage area. (ha)

If runoff happens, then this released P from manure water extractable P pools are carried away by runoff, the concentration of released P in runoff water depends upon phosphorus distribution factor (PDFACTOR) and it is calculated (Vadas et al., 2005) as

$$\text{PDFACTOR} = \left(\frac{\text{Runoff}}{\text{Rain}}\right)^{0.225} \quad (70)$$

Where, Runoff = Runoff amount. (cm)

Rain = Rainfall amount. (cm)

Manure P concentration in runoff water is calculated as

$$\text{Manpcrunoff} = \frac{\text{Manprelease}}{\text{Rain} * \text{Area} * 100} * \text{PDFACTOR} \quad (71)$$

Where, Manpcrunoff = Manure P concentration in runoff. (Kg/m^3)

Manure P mass loss through runoff is calculated as

$$\text{Manpmrunoff} = \text{Manpconrunoff} * \text{Runoff} * \text{Area} * 100 \quad (72)$$

Where, Manpmrunoff = Manure P mass loss through runoff. (Kg)

Area = Area of field. (ha)

The manure P which is released from manure water extractable P pools but not carried through runoff is absorbed to soil labile p pools. 60% of it is added to labile p pool of the first soil layer. In case of the depth of 2nd soil layer is more than 15cm then remaining 40% is added to the labile P pools of 2nd soil layer. If the depth of 2nd soil layer is less than 15cm then 30% of it added to labile pool of the 2nd soil layer and 10% is added to soil labile pool of 3rd soil layer.

$$\text{Lab}_a(1) = \text{Lab}_b(1) + (\text{Manprelease} - \text{Manpmassrunoff}) * 0.6 \quad (73)$$

If the depth of the second soil layer is more than 15 cm

$$\text{Lab}_a(2) = \text{Lab}_b(2) + (\text{Manprelease} - \text{Manpmassrunoff}) * 0.4 \quad (74)$$

If the depth of the second soil layer is less than 15 cm

$$\text{Lab}_a(2) = \text{Lab}_b(2) + (\text{Manprelease} - \text{Manpmassrunoff}) * 0.3 \quad (75)$$

$$\text{Lab}_a(3) = \text{Lab}_b(3) + (\text{Manprelease} - \text{Manpmassrunoff}) * 0.1 \quad (76)$$

4. Soil P Dynamics

There are constant sorption and desorption of P among the soil inorganic P pools (Figure 1) in order to maintain an equilibrium amount the inorganic P pools. A rapid sorption and

desorption exists between labile and active p pool, this is simulated based on Jones et al., (1984), with advance dynamic absorption and desorption (Vadas et al., 2006). The absorption and desorption of P between labile P and active P pool is depends upon P sorption coefficient (PSP) (Williams et al., 2008) and it is calculated as

$$PSP = -0.045 * \text{Log}(\text{Clay}) + 0.001 * \text{LabP} - 0.035 * \text{SoilOC} + 0.43 \quad (77)$$

An equilibrium is maintained between labile P and active P pool until the PBAL as defined by f equation 78 is zero. When PBAL >0, P from labile P pool moved to active P pool and when PBAL <0, P from active P pool moves to labile p pool.

$$PBAL = \text{LabP} - \text{ActP} * \frac{PSP}{1-PSP} \quad (78)$$

The movement of P from labile P pool to active P pool is calculated as i.e. when PBAL > 0

$$P\text{Flow}_{\text{lab} \rightarrow \text{Act}} = P_{\text{srpf}} * PBAL \quad (79)$$

The movement of P from active P pool to labile P pool is calculated as i.e. when PBAL < 0

$$P\text{Flow}_{\text{act} \rightarrow \text{lab}} = P_{\text{dsrpf}} * |PBAL| \quad (80)$$

Where, PBAL = A variable as defined by equation no 78. (Kg/ha)

$P\text{flow}_{\text{lab} \rightarrow \text{act}}$ = P flow from labile P pool to active P pool. (Kg/ha)

$P\text{flow}_{\text{act} \rightarrow \text{lab}}$ = P flow from active P pool to labile P pool (Kg/ha)

P_{srpf} = P sorption factor (-)

P_{dsrpf} = P desorption factor (-)

P_{srpf} and P_{dsrpf} dynamically changes daily as follows

$$P_{\text{srpf}} = A * \text{Day}^B \quad (81)$$

$$A = 0.918 * e^{-4.603 * PSP} \quad (82)$$

$$B = -0.238 * \text{Ln}(A) - 1.126 \quad (83)$$

Where, A = A factor as calculated by equation no 82

B = A factor as calculated by equation no 83

Day = Cumulative day since when the P in labile p pool increased and created an imbalance with active P pool and P movement from labile P pool to active p pool starts.

$$P_{\text{dsrpf}} = \text{Base} * \text{Day}^{-0.29} \quad (84)$$

$$\text{Base} = -1.08 * PSP + 0.79 \quad (85)$$

Where, Base = A factor as defined by equation 85.

Day = Cumulative day since when P in active P pool increased and created an imbalance with labile p pool and P movement from active P pool to labile P pool starts.

Similarly a slow absorption and desorption happens between P in active P pool and stable inorganic P pool. An equilibrium between active P pool and stable inorganic P pool is maintained as long as PBAL1 as defined by equation 86 is zero. When PBAL1>0, P from active P pools moved to stable inorganic P pool and when PBAL1<0, P from stable inorganic P pool moved to active P pool.

$$PBAL1 = 4 * ActP - StabiP \quad (86)$$

P flow from active P pools moved to stable inorganic P pool i.e. when PBAL1>0 is calculated as

$$Pflow_{actp \rightarrow stabip} = 0.0006 * PBAL1 \quad (87)$$

P flow from stable inorganic P pool pools moved to active P i.e. when PBAL1<0 is calculated as

$$Pflow_{stabip \rightarrow actp} = 0.00006 * |PBAL1| \quad (88)$$

Where, $Pflow_{actp \rightarrow Stabip}$ = P flow from active P to stable inorganic P pool. (Kg/ha)

$Pflow_{stabip \rightarrow actp}$ = P flow from stable inorganic P pool to active P pool. (Kg/ha)

PBAL1 = A variable as defined by equation 86. (Kg/ha)

Mineralization happens from fresh organic P pool and 80% of this mineralized P is added to soil labile P pool and remaining 20% is added to stable organic P pool (Jones et al., 1984). Mineralization is calculated as

$$Frsopmin = Frsominr * Frsop \quad (89)$$

Where, Frsopmin = Fresh organic P mineralization (Kg/ha/day)

Frsopminr = Fresh organic P mineralization rate (per day)

Frsop = Fresh organic P Pool. (Kg/ha)

$$Frsopminr = K_{or} * \sqrt{\gamma_{temp} \gamma_{water}} * \gamma_{ntr}$$

$$K_{or} = 0.8 \quad \text{when} \quad \frac{Crpres}{Crpres_i} > 0.8$$

$$= 0.05 \quad \text{when} \quad 0.1 < \frac{Crpres}{Crpres_i} < 0.8$$

$$= 0.0095 \quad \text{when} \quad \frac{Crpres}{Crpres_i} < 0.1 \quad (90)$$

Where, K_{or} = Rate constant (per day)

Mineralization also happens from stable organic P pool, and all the mineralized P from stable organic P pool is added to the labile P pool (Jones et al., 1984). Mineralization from stable organic P pool is calculated as

$$Stabopmin = K_{os} * MIN(\gamma_{temp} * \gamma_{water}) * Stabop \quad (91)$$

Where, Stabopmin = Mineralization from stable organic P pool. (Kg/ha/day)

K_{os} = rate constant of stable organic P mineralization =0.0003 per day

Stabop = P in stable organic P pool. (Kg/ha)

Immobilization happens from labile P pool. The immobilized P from labile P pool is added to soil fresh organic P pool (Jones et al., 1984). Immobilization from labile P pool is calculated as

$$Labpimmo = 0.16 * R_{or} * \frac{P_m}{O_m} \quad (92)$$

Where Labpimmo = Immobilization from labile P pool to fresh organic P pool. (Kg/ha/day)

P_m/O_m = It depends on Labile P amount and it varies between 0.01 and 0.02.

$$\begin{aligned} \frac{P_m}{O_m} &= 0.02 \text{ if } LabP > 10 \\ &= 0.01 + 0.001 * Labp \quad \text{if } LabP < 10 \end{aligned} \quad (93)$$

R_{or} = Immobilization rate (per day)

$$R_{or} = K_{or} * Crpres * \sqrt{\gamma_{temp} * \gamma_{water}} * \gamma_{ntr} \quad (94)$$

5. Plant P Uptake

Fraction of P in the plant biomass on a given day in case of optimal plant growth is calculated as

$$f_{rp} = (f_{rp1} - f_{rp3}) \left[1 - \frac{f_{rphu}}{f_{rphu} + \exp(P_1 - P_2 \cdot f_{rphu})} \right] + f_{rp3} \quad (95)$$

Where, f_{rp} = Fraction of P in plant biomass on a given day, in case of optimal plant growth

f_{rp1} = Normal fraction of P in plant biomass at emergence.

f_{rp3} = Normal fraction of P in plant biomass at maturity.

f_{rphu} = Fraction of potential heat unit (PHU) accumulated for the plant on a given day.

P_1, P_2 are shape coefficients. These are calculated as follows

$$P_1 = \ln \left[\frac{\frac{f_{rphu,50\%}}{1 - \frac{f_{rp2} - f_{rp3}}{f_{rp1} - f_{rp3}}} - f_{rphu,50\%}}{f_{rphu,50\%}} \right] + P_2 \cdot f_{rphu,50\%} \quad (96)$$

$$P_2 = \frac{\ln \left[\frac{\frac{f_{rphu,50\%}}{1 - \frac{f_{rp2} - f_{rp3}}{f_{rp1} - f_{rp3}}} - f_{rphu,50\%}}{f_{rphu,50\%}} \right] - \ln \left[\frac{\frac{f_{rphu,100\%}}{1 - \frac{f_{rp3} - f_{rp3}}{f_{rp1} - f_{rp3}}} - f_{rphu,100\%}}{f_{rphu,100\%} - f_{rphu,50\%}} \right]}{f_{rphu,100\%} - f_{rphu,50\%}} \quad (97)$$

Where, f_{rp2} = Normal fraction of P in plant biomass at 50% maturity.

f_{rp3} = Normal fraction of P in plant biomass at near maturity.

$f_{rphu, 50\%}$ = Fraction of potential heat unit (PHU) accumulated for the plant at 50% maturity.

$$= 0.5$$

$f_{rphu, 100\%}$ = Fraction of potential heat unit (PHU) accumulated for the plant at 100% maturity.

$$= 1$$

Model assumes the value of $(f_{rp3} - f_{rp3}) = 0.0001$, in order to avoid the second ln term in the equation 105 to become undermined.

Optimal mass of P that should be stored in plant biomass on a given day is calculated as

$$Bio_{p,opt} = f_{rp} * Bio \quad (98)$$

Where, $Bio_{p, opt}$ = Optimum mass of P that should be stored in plant biomass on a given day. (Kg/ha)

Bio = Total plant biomass of a given day. (Kg/ha)

Plant P demand for a day is calculated as

$$P_{demand} = 1.5 * MIN(Bio_{p,opt} - Bio_p, 4 \cdot f_{rp3} \cdot \Delta Bio) \quad (99)$$

Where, P_{demand} = Plant P demand on a given day. (Kg/ha)

Bio_p = Actual Plant P in a given day. (Kg/ha)

ΔBio = Potential increase in total plant biomass on a given day. (Kg/ha)

The P_{up} amount of P is up taken by the plant from the soil labile P pool only. The depth distribution of plant P uptake is calculated as

$$P_{up,z} = \frac{P_{demand}}{1 - \exp(-\beta_p)} \left[1 - \exp(\beta_p \frac{z}{z_{root}}) \right] \quad (100)$$

Where, $P_{up,z}$ = Potential P uptake from the soil surface to depth z (Kg/ha)

β_p = Plant P uptake distribution parameter.

z = Depth from soil surface. (m)

z_{root} = Depth of root in soil from the soil surface on a given day. (m)

The potential P uptake from a soil layer is calculated by

$$P_{up,ly} = P_{up,zl} - P_{up,zu} \quad (101)$$

Where, $P_{up, ly}$ = Potential P uptake from a soil layer. (Kg/ha)

$P_{up, zl}$ = Potential P uptake from soil surface to the lower boundary of the soil layer. (Kg/ha)

$P_{up, zu}$ = Potential P uptake from soil surface to the upper boundary of the soil layer. (Kg/ha)

Finally, the actual P uptake by the plant from a soil layer is calculated as

$$P_{act,ly} = MIN(P_{up,ly}, P_{demand}, LabP_{ly}) \quad (102)$$

Where, $P_{act, ly}$ = Actual P uptake by the plant from a layer. (Kg/ha)

P_{demand} = P uptake demand not met by the overlaying soil layers. (Kg/ha)

$\text{LabP}_{ly} = \text{P in labile P pool of the layer. (Kg/ha)}$

If the all the P demand of the crop can't not be met by soil labile P pool then, the plant under goes P stress and the yield get reduced. P stress is calculated as

$$P_{\text{stress}} = 1 - \frac{\varphi_p}{\varphi_p + \exp(3.535 - 0.02597\varphi_p)} \quad (103)$$

Where, $P_{\text{stress}} = \text{P stress for a given day.}$

$\varphi_p = \text{Scaling factor for P stress and it is calculated as}$

$$\varphi_p = 200 \cdot \left[\frac{\text{Bio}_p}{\text{Bio}_{p,\text{opt}}} - 0.05 \right] \quad (104)$$

6. P flow from Residue and Soil Organic matter

To simulate the P flow from the crop residue and soil organic matter the crop residues and soil organic matter are divided into five computational P pools as described within the soil nutrient module of the RZWQM2 model (Ma et al., 2012). The crop residue is divided into two pools namely fast residue pool and slow residue pool whereas Soil organic matter is divided into three pools namely fast organic matter pool, intermediate organic matter pool and slow organic matter pool. At beginning of the simulation P mass in these pools are initialized using the user defined initial C: P ratio. The P in these pools are decomposed daily at the same rate of carbon decomposition as computed (Rojas and Hanson, 2000) by the RZWQM2 model P is transferred within pools as shown in Figure 2. P released due to degradation of residue pools will be added to the Fresh Organic P pool and P released due to the degradation of soil organic matter pools is added to stable organic P pool.

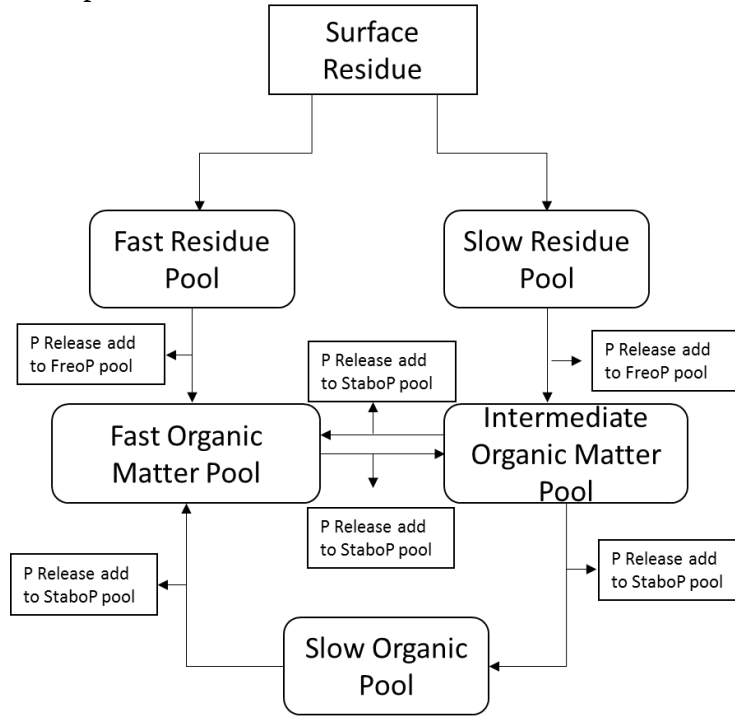


Figure 2: P low from Residue and Soil Organic mater

7. Tillage

Tillage operation incorporates the surface P pools i.e. fertilizer P pools and manure P pools into soil based on tillage incorporation efficiency and mixes the soil P pools and crop residues based on tillage mixing efficiency, tillage depth and ratio of soil mass of a layer to total soil mass of layers up to tillage depth. During tillage operation fertilizer P pools and manure water extractable P pools incorporates into soil labile P pool whereas the manure stable P pools incorporates into the active P pool of the first soil layer.

$$LabP_a(1) = LabP_b(1) + (Avfertp + Resfertp + Manwip + Manwop) * \frac{Tillinceffi}{100} \quad (105)$$

$$ActP_a(1) = ActP_b(1) + (Mansip + Mansop) * \frac{Tinceffi}{100} \quad (106)$$

Where, $LabP_a(1)$ = Labile P of the first soil layer after the incorporation due to tillage. (Kg)

$LabP_b(1)$ = Labile P of the first soil layer before the incorporation due to tillage. (Kg)

$ActP_a(1)$ = Active P of the first soil layer after the incorporation due to tillage. (Kg)

$ActP_b(1)$ = Active P of the first soil layer before the incorporation due to tillage. (Kg)

$Tinceffi$ = Tillage incorporation efficiency. (%)

Tillage operation also mixes the soil P pools and crop residues of all the layers having depth less than tillage depth as follows

$$LabP_a = \left(1 - \frac{Tmixeffi}{100}\right) LabP_b + TlabP * Soil_{ratio} * \frac{Tmixeffi}{100} \quad (107)$$

$$ActP_a = \left(1 - \frac{Tmixeffi}{100}\right) ActP_b + TActP * Soil_{ratio} * \frac{Tmixeffi}{100} \quad (108)$$

$$StabiP_a = \left(1 - \frac{Tmixeffi}{100}\right) StabiP_b + TStabiP * Soil_{ratio} * \frac{Tmixeffi}{100} \quad (109)$$

$$StaboP_a = \left(1 - \frac{Tmixeffi}{100}\right) StaboP_b + TStaboP * Soil_{ratio} * \frac{Tmixeffi}{100} \quad (110)$$

$$FrsoP_a = \left(1 - \frac{Tmixeffi}{100}\right) FrsoP_b + TFrsoP * Soil_{ratio} * \frac{Tmixeffi}{100} \quad (111)$$

Where, $LabP_a$ = Labile P pool after the mixing due to tillage. (Kg)

$LabP_b$ = Labile P pool before the mixing due to tillage. (Kg)

$ActP_a$ = Active P pool after the mixing due to tillage. (Kg)

$ActP_b$ = Active P pool before the mixing due to tillage. (Kg)

$StabiP_a$ = Stabip P pool after the mixing due to tillage. (Kg)

$StabiP_b$ = Stabip P pool before the mixing due to tillage. (Kg)

$StaboP_a$ = Stabop P pool after the mixing due to tillage. (Kg)

$StaboP_b$ = Stabop P pool before the mixing due to tillage. (Kg)

$FrsoP_a$ = Frsop P pool after the mixing due to tillage. (Kg)

$FrsoP_b$ = Frsop P pool before the mixing due to tillage. (Kg)

$TLabP$ = Sum of the P of all the soil labile P pool for the layers having depth less than tillage depth. (Kg)

$TActp$ = Sum of the P of all the soil active P pool for the layers having depth less than tillage depth. (Kg)

$TStabiP$ = Sum of the P of all the soil Stabip P pool for the layers having depth less than tillage depth. (Kg)

$TStabop$ = Sum of the P of all the soil Stabop P pool for the layers having depth less than tillage depth. (Kg)

$TFrsop$ = Sum of the P of all the soil Stabop P pool for the layers having depth less than tillage depth. (Kg)

$Soil_{ratio}$ = The ratio of soil mass of a layer to the total soil mass of the layers having depth less than tillage depth. (Kg)

$Tmixeffi$ = Tillage mixing efficiency. (%)

8. DRP loss Runoff

Dissolve reactive P (DRP) loss through runoff is calculated as

$$Drplossrnf = Fertpmrunoff + Manpmrunoff + Labpmrunoff \quad (112)$$

Where, $Drplossrnf$ = Amount of DRP loss through runoff. (Kg/ha)

$Fertpmrunoff$ = Fertilizer P loss through runoff. (Kg/ha) [Chapter 3]

$Manpmrunoff$ = Manure P loss through runoff. (Kg/ha) [Chapter 4]

$Labpmrunoff$ = Soil labile P loss through runoff. (Kg/ha)

$Labpmrunoff$ is calculated as

$$Labpmrunoff = \frac{LabP(1) * Runoff}{\rho_{b1} * Dsoil(1) * K_{d1}} \quad (113)$$

Where, $Labp(1)$ = P in labile P pool of the first soil layer. (Kg/ha)

$Runoff$ = Amount of surface runoff on a given day. (m)

ρ_{b1} = Bulk density of the first soil layer. (Kg/m³)

$Dsoil(1)$ = Depth of the first soil layer. (m)

K_{d1} = Soil partitioning coefficient of the first soil layer (m³/Kg)

Soil partitioning coefficient is depend on fraction of clay content of soil and is calculated as

$$K_{d1} = 0.1 + 0.25 \text{ clay}(1) \quad (114)$$

Where, $\text{clay}(1)$ = Fraction of clay content of the first soil layer. (-)

9. PP loss Runoff

Particulate Phosphorus (PP) loss through surface runoff is calculated as

$$PPloss_{rnf} = 0.001 * Conc_{sedp} * \frac{sed}{Area} * Encr \quad (115)$$

Where, $PPloss_{rnf}$ = Amount of PP loss through runoff (Kg/ha)

$Conc_{sedp}$ = Concentration of P attached to sediment in the surface soil layer. (gm P/MT soil)

Sed = Sediment yield on a given day. (MT)

Area = Area of the field. (ha)

Encr = P enrichment ratio (-)

$$Conc_{sedp} = 100 \frac{LabP(1) + ActP(1) + Stabip(1) + Stabop(1) + Frsop(1) + Avfertp + Resferp + Manwip + Manwop + Mansop + Mansip}{\rho_{b1} * Dsoil(1)} \quad (116)$$

$$Encr = 0.78 * Conc_{sed,rnf}^{-0.2468} \quad (117)$$

Where,

Encr = P enrichment ratio (-)

$Conc_{sed, rnf}$ = Concentration of sediment in runoff (Mg/m³)

$$Conc_{sed,rnf} = \frac{sed}{10 * Area * runoff} \quad (118)$$

Where, Sed = Sediment yield on a given day. (MT)

Area = Area of the field. (ha)

Runoff= Amount of surface runoff on a given day. (mm)

10. DRP Loss Tile Drainage

To simulate DRP loss through tile drainage linear groundwater reservoir based approach as suggested by TAM-MO-DEL (Steenhuis et al., 1997) is used. In this approach DRP through matrix flow and as well as macropore flow at first contributes to the groundwater reservoir, from which then DRP is lost along with the drainage water.

Amount of P leached out from a layer by matrix flow is calculated as (Francesconi et al., 2016)

$$P_{Leach,mat} = C_{labp,sw} \left(1 - \exp \left(\frac{-q_{mat}}{K_d m_s + SW} \right) \right) \quad (119)$$

Where, $P_{leach, mat}$ = Amount of DRP loss through matrix flow from a soil layer. (Kg/m³)

$C_{labp, sw}$ = Concentration of Labp is soil water in layer. (Kg/m³)

q_{mat} = Amount of matrix flow percolating out of a soil layer. (m)

K_d = Soil partitioning coefficient (m³/Kg)

m_s = Mass of a soil layer. (Kg/m²)

SW = Soil water content of the soil layer. (m)

$$C_{labp,SW} = \frac{LabP}{K_d m_s + SW} \quad (120)$$

$$K_d = 0.1 + 0.25 \text{ clay} \quad (121)$$

Where, $Labp$ = P in labile P pool of a soil layer. (Kg/m²)

Clay = Clay fraction of a soil layer. (-)

Amount of DRP loss from first soil layer is added to labile P pool of the next soil layer and so on until it reaches the groundwater reservoir and added to it.

In case of DRP loss through macropore flow, It is assumed that macropore flow occurs as a short-circuit flow i.e. it is originated from the first soil layer and directly contributes to the groundwater reservoir. Amount of P leached out by the macropore flow is calculated as (Steenhuis et al., 1994).

$$P_{leach,mac} = C_{labp,SW,1} \left(1 - \exp \left(\frac{-R}{K_{d1} m_{s1} + SW_1} \right) \right) * r \quad (122)$$

Where, $P_{leach, mac}$ = Amount of DRP loss through macropore flow. (Kg/m³)

R = Rainfall amount (m)

r = ratio of macropore flow to the total flow from the first soil layer. (-)

$$r = \frac{V_{mac}}{V_{mac} + V_{mat}} \quad (123)$$

Where, V_{mac} = Volume of macropore flow. (m³)

V_{mat} = Volume of matrix flow. (m³)

1 in subscripts stands for all the variables as defined above for the first soil layer only.

DRP loss from the groundwater reservoir is calculated by mass balance approach. i.e.

Change in P mass in GW reservoir = Incoming P mass – Outgoing P mass

$$y'(t) = I_{drp} - \frac{y(t)}{S_{gw}} * drain \quad (124)$$

Where, $y(t)$ = Mass of DRP present at any time t in the groundwater reservoir. (Kg)

I_{drp} = Incoming DRP mass to groundwater reservoir through macropore and matrix flow. (Kg)

S_{gw} = Storage volume of the groundwater reservoir during time t . (m³)

Drain = Outflow volume from the groundwater reservoir i.e. the tile drainage amount. (m³)

t = time. (days)

By solving equation 124 we get

$$y(t) = \frac{S_{gw} * I_{drp}}{drain} + \left(y_0 - \frac{S_{gw} * I_{drp}}{drain} \right) * \exp\left(-\frac{drain}{S_{gw}} * t\right) \quad (125)$$

In case of Drain = 0 then

$$y(t) = I_{drp} * t + y_0 \quad (126)$$

Where, y_0 = initial amount of P mass in the groundwater reservoir at the beginning of the day.
(Kg)

Average concentration of DRP in 1 day the groundwater reservoir

$$C_{drp,gw} = \frac{y_0 + y(1)}{2 * S_{gw}} \quad (127)$$

Where, $C_{drp,gw}$ = Concentration of DRP in groundwater reservoir. (Kg/m³)

The mass of DRP loss through tile drainage is calculated as

$$Drp_{losstdrain} = C_{drp,gw} * drain \quad (128)$$

Where, $Drp_{losstdrain}$ = Mass of DRP loss through tile drainage. (Kg)

11. PP Loss Tile Drainage

PP loss through tile drainage is based on the model described by Jarvis et al., 1999. In this approach it is assumed that PP loss through tile drainage only happens through macropore flow and originates only from the first soil layer. The macropore flow along PP is added to the groundwater reservoir then it finally loss through tile drainage with drainage water.

Soil detachment is calculated as

$$D = K_d * E * R * M_s * Crop \quad (129)$$

Where, D = Detachment of soil particle. (gm m⁻² day⁻¹)

K_d = Soil detachability coefficient. (gm J⁻¹)

E = Kinetic energy of the rain. (J m⁻² mm⁻¹)

R = Rainfall rate. (mm day⁻¹)

M_s = Amount of readily available dispersible particle (gm gm⁻¹ soil)

Crop = An empirical crop management factor used in USLE for reduction in particle detachment when the crop covers the soil.

$$Crop = 1 - FC * \exp(-0.34H) \quad (130)$$

Where, H = effective canopy height (m)

FC = fraction of land surface covered by crop canopy. (-)

$$H = 0.6 * \text{crop height} \quad (131)$$

$$FC = 6.5 * LAI^{0.75} * S^{-0.48} \quad (132)$$

Where, LAI = leaf area index. (-)

S = row spacing. (mm)

E depends upon the amount of rainfall and it is calculated as

$$E = 29 * (1 - 0.72 \exp(-0.05R)) \quad (133)$$

The value of M_s dynamically changes due to particle replenishment. In case of just after tillage, if the M_s does not reached 50% of its maximum value (M_{smax}) then the model make it 50% of its maximum value. M_{smax} is calculated as

$$M_{smax} = 0.362 * clay - 0.518 \quad (134)$$

Where clay = clay content of the soil (%)

If we do mass balance of the available particle at the soil surface (A_s) then

$$\frac{dA_s}{dt} = -D + P \quad (135)$$

Where, P is the particle replenishment. It is calculated as

$$P = K_r \left(1 - \frac{M_s}{M_{smax}}\right) \quad (136)$$

Where, K_r = Replenishment rate coefficient. ($\text{gm m}^{-2} \text{day}^{-1}$)

$$A_s = M_s * \gamma * Z_d \quad (137)$$

Where, γ = Bulk density of the surface soil layer. (gm/m^3)

Z_d = Depth of the surface soil layer. (m)

Now, substituting the value of P, A_s and D in the equation 135 then solving it for M_s we get

$$M_s = \frac{1}{K} \left(K_r + (KM_{so} - K_r) * \exp\left(-\frac{Kt}{\gamma Z_d}\right) \right) \quad (138)$$

$$K = K_d * E * R * Crop + \frac{K_r}{M_{smax}} \quad (139)$$

Where, M_{so} = Initial amount of M_s before the beginning of a day.

t = time (1 day)

Concentration of suspended particle routed into the macropore flow is calculated as

$$C_{sp,mac} = \frac{D}{R + Z_d SW_1} \quad (140)$$

Where, $C_{sp,mac}$ = Concentration of suspended particle routed into the macropore flow

($\text{gm m}^{-2} \text{mm}^{-1}$)

R = Rainfall (mm)

Z_d = Depth of the surface soil layer. (mm)

SW_1 = Soil water content of the surface soil layer. (m^3/m^3)

Due to filtering, the mass of suspended particle reaching the groundwater reservoir decreases as follows

$$M_d = q_{mac} * C_{sp.mac} * \exp(-f * d_{gw}) \quad (141)$$

Where M_d = Mass of suspended particle reaching the groundwater reservoir. (gm m^{-2})

f = filter coefficient (m^{-1})

d_{gw} = Depth to groundwater reservoir. (m)

Mass of PP reaching groundwater reservoir is calculated as

$$M_{pp} = f_{md} * (Labp(1) + ActP(1) + Stabip(1) + Frsop(1) + Stabop(1) + Manwip + Manwop + Mansip + Mansop + Avfertp + Resfertp) \quad (142)$$

Where, M_{pp} = Mass of PP reaching groundwater reservoir (Kg/ha)

f_{md} = A factor

$$f_{md} = \frac{M_d}{\gamma Z_d} \quad (143)$$

Where, γ = Bulk density of the first soil layer. (gm/m^3)

Z_d = Depth of the surface soil layer. (m)

PP loss from groundwater reservoir through tile drainage is calculated by mass balance approach
Change in PP mass in groundwater reservoir = Incoming PP mass – Outgoing PP mass i.e.

$$\frac{dy}{dt} = I_{pp} - \frac{y(t)}{S_{gw}} \text{drain} \quad (144)$$

Where, $y(t)$ = Mass of PP present at any time t in the groundwater reservoir. (Kg)

I_{pp} = Incoming PP mass to groundwater reservoir through macropore flow. (Kg)

S_{gw} = Storage volume of the groundwater reservoir during time t . (m^3)

Drain = Outflow volume from the groundwater reservoir i.e. the tile drainage amount. (m^3)

t = time. (days)

Solving equation 144 we get

$$y(t) = \frac{S_{gw}}{\text{drain}} * I_{pp} + \left(y_0 - \frac{S_{gw}}{\text{drain}} * I_{pp} \right) * \exp\left(-\frac{\text{drain}}{S_{gw}} * t\right) \quad (145)$$

In case of drain =0

$$y(t) = I_{pp} * t + y_0 \quad (146)$$

Where, y_0 = initial amount of PP mas in the groundwater reservoir at beginning of the day. (Kg)

Concentration of PP in groundwater reservoir in a day is calculated as

$$C_{pp,Gw} = \frac{y_0 + y(1)}{2 * S_{gw}} \quad (147)$$

Where, $C_{pp,gw}$ = Concentration of PP in groundwater reservoir. (Kg/m^3)

Amount of PP loss through tile drainage is calculated as

$$PP_{losstdrain} = C_{pp,gw} * drain \quad (148)$$

Where, $PP_{losstdrain}$ = Mass of PP loss through tile drainage. (Kg)

12. References

- Ajiboye, B., Akinremi, O., & Racz, G. (2004). Laboratory characterization of phosphorus in fresh and oven-dried organic amendments. *J. Environ Qual*, 33(3), 1062-1069.
- Dou, Z., Toth, J., Galligan, D., Ramberg, C., & Ferguson, J. (2000). Laboratory procedures for characterizing manure phosphorus. *J. Environ Qual*, 29(2), 508-514.
- Francesconi, W., Williams, C., Smith, D., Williams, J., & Jeong, J. (2016). Phosphorus Modeling in Tile Drained Agricultural Systems Using APEX. *J Fertil Pestic*, 7(166), 2-10.
- He, Z., Honeycutt, C., & Griffin, T. (2003). Comparative investigation of sequentially extracted phosphorus fractions in a sandy loam soil and a swine manure. *Commun Soil Sci Plan*, 34(11-12), 1729-1742.
- He, Z., & Honeycutt, C. W. (2001). Enzymatic characterization of organic phosphorus in animal manure. *J. Environ Qual*, 30(5), 1685-1692.
- He, Z., Senwo, Z. N., Mankolo, R. N., & Honeycutt, C. W. (2006). Phosphorus fractions in poultry litter characterized by sequential fractionation coupled with phosphatase hydrolysis. *J. Food Agric Environ*, 4(1), 304-310.
- Jarvis, N., Villholth, K., & Ulen, B. (1999). Modelling particle mobilization and leaching in macroporous soil. *Eur J. Soil Sci*, 50(4), 621-632.
- Jones, C., Cole, C., Sharpley, A., & Williams, J. (1984). A simplified soil and plant phosphorus model: I. Documentation. *Soil Sci Soc Am J.*, 48(4), 800-805.
- Ma, L., Ahuja, L., Nolan, B., Malone, R., Trout, T., & Qi, Z. (2012). Root Zone Water Quality Model (RZWQM 2): Model use, calibration, and validation. *Trans. of the ASABE*, 55(4), 1425-1446.

- McDowell, R., & Stewart, I. (2005). Phosphorus in fresh and dry dung of grazing dairy cattle, deer, and sheep. *J. Environ Qual*, 34(2), 598-607.
- McGrath, J. M., Sims, J. T., Maguire, R. O., Saylor, W. W., Angel, C. R., & Turner, B. L. (2005). Broiler diet modification and litter storage. *J. Environ Qual*, 34(5), 1896-1909.
- Rojas, K., & Hanson, J. (2000). Root zone water quality model: modelling management effects on water quality and crop production: Water Res Pub.
- Steenhuis, T. S., Bodnar, M., Geohring, L. D., Aburime, S.-A., & Wallach, R. (1997). A simple model for predicting solute concentration in agricultural tile lines shortly after application. *Hydrol Earth Syst Sci*, 1(4), 823-833.
- Steenhuis, T. S., Boll, J., Shalit, G., Selker, J. S., & Merwin, I. A. (1994). A simple equation for predicting preferential flow solute concentrations. *J. Environ Qual*, 23(5), 1058-1064.
- Turner, B. L., & Leytem, A. B. (2004). Phosphorus compounds in sequential extracts of animal manures: Chemical speciation and a novel fractionation procedure. *Environ Sci Technol*, 38(22), 6101-6108.
- Vadas, P., Gburek, W., Sharpley, A., Kleinman, P., Moore, P., Cabrera, M., & Harmel, R. (2007). A model for phosphorus transformation and runoff loss for surface-applied manures. *J. Environ Qual*, 36(1), 324-332.
- Vadas, P., Haggard, B., & Gburek, W. (2005). Predicting dissolved phosphorus in runoff from manured field plots. *J. Environ Qual*, 34(4), 1347-1353.
- Vadas, P., Kleinman, P., & Sharpley, A. (2004). A simple method to predict dissolved phosphorus in runoff from surface-applied manures. *J. Environ Qual*, 33(2), 749-756.
- Vadas, P., Krogstad, T., & Sharpley, A. (2006). Modeling Phosphorus Transfer between Labile and Nonlabile Soil Pools. *Soil Sci Soc Am J*. 70(3), 736-743.
- Vadas, P. A. (2014). Surface Phosphorus and Runoff Model, Theoretical Documentation Version 1.0.
- Vadas, P. A., Owens, L. B., & Sharpley, A. N. (2008). An empirical model for dissolved phosphorus in runoff from surface-applied fertilizers. *Agric. Ecosyst. Environ.* 127(1-2), 59-65.
- Williams, C. (1969). Moisture uptake by surface-applied superphosphate and movement of the phosphate and sulphate into the soil. *Soil Res.* 7(3), 307-316.
- Williams, J., R. Izaurrealde and E. Steglich. (2008). Agricultural policy/environmental extender model theoretical documentation. Temple, Texas, US: Blackland research and Extension Center.